B.Sc. 3rd Semester (Honours) Examination, 2019-20

MATHEMATICS

Course ID : 32114 **Course Code : SHMTH-304/GE-3**

Course Title: Algebra

Time: 2 Hours **Full Marks: 40**

The figures in the right hand side margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Unless otherwise mentioned, notations and symbols have their usual meaning.

1. Answer *any five* questions: 2×5=10

(a) If $Z + \frac{1}{Z} = \sqrt{3}$, then show that $Z^6 = -1$.

(b) Show that $n^n \geq n!$ for all $n \in N$.

(c) Give an example of transitive relation which is

(i) symmetric but not reflexive and

(ii) reflexive but not symmetric.

(d) Transform the equation to remove the square from $x^3 - 15x^2 - 33x + 84 = 0$.

(e) Apply Descartes rule of signs to examine the nature of the roots of the equation $x^4 + 2x^2 + 3x - 1 = 0$.

(f) Use division algorithm, find integers u and v satisfying $63u + 55v = 1$

(g) Find the dimension of the subspace W of $\mathbb{R}^4$, where

$W = \{(x, y, z, w) \in \mathbb{R}^4 : x + 2y - z = 0,\ 2x + y + w = 0\}$

(h) Define $T: \mathbb{R}^2 \to \mathbb{R}^2$ by $T(x, y) = (2x + y, y)$, $(x, y) \in \mathbb{R}^2$ Examine whether T is linear or not? Justify your answer.

2. Answer *any four* questions: 5×4=20

(a) (i) Show that the product of all values of $\left(1 + i\sqrt{3}\right)^{\frac{3}{4}}$ is 8.

(ii) State Cauchy-Schwarz inequality. 3+2=5

(b) Define linear transformation. Find out the matrix corresponding to the linear transformation $T: \mathbb{R}^3 \to \mathbb{R}^2$ $s.t.$

$$T(x, y, z) = \left(2x + 2y + z, \frac{-x + y + 3z}{2}\right)$$

with the respect to the ordered basis $\{(0, 1, 1)(1, 0, 1)(1, 1, 0)\}$ of $\mathbb{R}^3$ and $\{(1, 0), (1, 1)\}$ of $\mathbb{R}^2$.

(c) Find the eigenvalues and eigenvectors of the matrix:

$$\begin{pmatrix} 1 & 1 & 1 \\ -1 & -1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$$

(d) (i) Find the product of all values of $(1+i\sqrt{3})^{\frac{3}{4}}$

(ii) If $x+\frac{1}{x}=2\cos\theta$ and θ is real, prove that $x^n+\frac{1}{x^n}=2\cos n\theta$. 3+2=5

(e) (i) Prove that the product of any m consecutive integers is divisible by m.

(ii) If a is prime to b, prove that a^2 is prime to b and a^2 is prime to b^2. 3+2=5

(f) Solve, if possible, the system of equations

$x_1+2x_2-x_3=10$

$-x_1+x_2+2x_3=2$

$2x_1+x_2-3x_3=8$

3. Answer *any one* question: 10×1=10

(a) (i) Find the remainder when $1!+2!+\cdots+50!$ is divided by 15.

(ii) If Z is a non-zero complex number and p, q, m and n are positive integers, where $\frac{p}{q}=\frac{m}{n}$ with $\gcd(m,n)=1$, then $Z^{\frac{p}{q}}=Z^{\frac{m}{n}}$.

(iii) Solve the equation using Cardan's method $x^3-18x-35=0$ 2+3+5=10

(b) (i) State Cayley-Hamilton theorem. Verify Cayley-Hamilton theorem for the matrix $A=\begin{pmatrix} 1 & 0 & 0 \\ 1 & 2 & 1 \\ 2 & 3 & 2 \end{pmatrix}$ and hence compute A^{-1}.

(ii) The matrix of a linear mapping $T:\mathbb{R}^3\to\mathbb{R}^2$ relative to the ordered bases $\{(0,1,1),(1,0,1),(1,1,0)\}$ of $\mathbb{R}^3$ and $\{(1,0),(1,1)\}$ of $\mathbb{R}^2$ is $\begin{pmatrix} 1 & 2 & 4 \\ 2 & 1 & 0 \end{pmatrix}$.Find the linear transformation T . Find the matrix of T relative to the ordered bases $\{(1,1,0),(1,0,1)(0,1,1)\}$ of $\mathbb{R}^3$ and $\{(1,1),(0,1)\}$ of $\mathbb{R}^2$. (1+3+1)+(3+2)=10

———